Africa Development, Vol. XXX, No.3, 2005, pp. 48–65

(ISSN 0850-3907)

Ethnomathematics, Geometry and Educational Experiences in Africa

Paulus Gerdes*

Abstract

The paper traces historically reflections about mathematics, education and culture in Africa, that culminated in the emergence of ethnomathematics as a research field. A brief overview of ethnomathematical research in Mozambique and of historical research related to mathematics in Africa is presented, followed by examples of the integration of ethnomathematics into teacher education to stimulate the development of social—and cultural—mathematical awareness. The paper concludes with some trends in using ideas from ethnomathematics in education in Africa.

Résumé

Cet article fait un recensement historique des réflexions qui ont été menées dans le domaine des mathématiques, de l'éducation et de la culture en Afrique, et qui ont favorisé l'émergence de l'ethnomathématique, comme champ de recherche. Ce texte présente un bref aperçu de la recherche en ethnomathématique au Mozambique, mais également de la recherche historique en mathématiques en Afrique. Puis, il présente quelques exemples d'intégration de l'ethno-mathématique dans l'éducation scolaire, censée favoriser la sensibilisation sociale et culturelle aux mathématiques. Cet article se termine sur quelques tendances montrant comment appliquer quelques principes d'ethnomathématique dans le domaine de l'éducation en Afrique.

* Dr. Paulus Gerdes is Professor of Mathematics. Director, ethnomathematics Research Centre, Maputo, Mozambique. E-mail: pgerdes@virconn.com.

Ethnomathematics is the relatively young field of research that started to emerge in the 1970s and 1980s among mathematics educators and researchers worried about the mathematical marginalisation of the peoples, in particular the poor, of the Third World and of people of African descent and other minorities in the First World (for an overview, see Gerdes 1996). The Brazilian Ubiratan D'Ambrosio, who also worked for UNESCO in Mali, and who visited Mozambique in 1978, is often called the 'father of ethnomathematics'. He proposed his ethnomathematical programme as a 'methodology to track and analyse the processes of generation, transmission, diffusion and institutionalisation of (mathematical) knowledge in diverse cultural systems' (D'Ambrosio 1990). In the context of the African continent several concepts had been proposed to underline the existence of mathematical ideas and practices in African cultures before and simultaneously with the transplantation of schooling and mathematics curricula from the so-called West to the continent:

- '*Indigenous mathematics*' (Cf. Gay & Cole 1967). Criticizing education of Kpelle children (Liberia) in 'western-oriented' schools – they 'are taught things that have no point or meaning within their culture' – Gay and Cole proposed a creative mathematical education that uses the indigenous mathematics as starting point;
- *Sociomathematics* of Africa (Zaslavsky 1973): 'the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of their mathematics';
- *Informal* mathematics (Posner 1978, 1982): mathematics that is transmitted and that one learns outside the formal system of education (referring to Côte d'Ivoire);
- *Mathematics in the socio-cultural environment* [S. Doumbia, S. Touré (Côte d'Ivoire) 1984]: integration of the mathematics of African games and craftwork that belongs to the social-cultural environment of the child into the mathematics curriculum;
- *Oral mathematics* (Kane 1987): in all human societies there exists mathematical knowledge that is transmitted orally from one generation to the next (Kane's doctoral dissertation studied numeration systems in West Africa);
- *Oppressed* mathematics (Gerdes 1982): in African countries there exist mathematical elements in the daily life of the populations, that have not been recognized as mathematics by the dominant (colonial and neo-colonial) ideologies;

- *Non-standard* mathematics (Gerdes 1982, 1985a): beyond the dominant standard forms of 'academic' and 'school' mathematics there has developed in all cultures mathematical forms that are distinct;
- *Hidden* or *frozen* mathematics (Gerdes 1982, 1985a, b): although, probably, most of mathematical knowledge of the formerly colonized peoples has been lost for ever, it is possible to reconstruct or 'unfreeze' some of the mathematical thinking, that is 'hidden' or 'frozen' in old techniques, like, e.g., that of basket making;
- *People's* mathematics as a component of people's education in the context of the struggle against apartheid in South Africa (Julie 1989);
- *Implicit* mathematics (Zaslavsky 1994).

These concept proposals were provisional. Some of them emerged also in cultural contexts outside Africa. The various aspects illuminated by these concepts have been gradually united under the more general 'common denominator' of D'Ambrosio's ethnomathematics. This process has been accelerated by the creation of the *International Study Group on Ethnomathematics* (ISGEm) in 1985.

An early forerunner of reflections about mathematics education and African indigenous cultures was Otto Raum. He published in 1938 the book '*Arithmetic in Africa*', based on his experiences in South Africa and Tanganyika.

Ethnomathematical research in Mozambique

In 1985 I concluded a study on culture and the awakening of geometrical thinking. The study reveals mathematical activity in diverse cultural practices. As most 'mathematical' traditions that survived colonisation and most 'mathematical' activities in daily life are not explicitly mathematical, i.e. the mathematics is partially 'hidden', the first aim of this research was to 'uncover' the 'hidden' mathematics. The first results of this 'uncovering' are included in book 'On the awakening of geometrical thinking' (Gerdes 1985b; cf. Gerdes 2003) and slightly extended in 'Ethnogeometry: cultural-anthropological contributions to the genesis and didactics of geometry' (Gerdes 1991a).

At the end of the 1980s it turned out to be possible to integrate into MERP some young, well-motivated Mozambicans. They had earned their M.Ed. in mathematics abroad, like Abdulcarimo Ismael, Marcos Cherinda and Daniel Soares, who after initial teacher education in Mozambique had continued their education in the Germany. Later they concluded doctoral theses in the field of ethnomathematics. Abdulcarimo Ismael's dissertation is entitled 'An ethnomathematical study of Tchadji - about a Mancala type board game played in Mozambique and possibilities for its use in Mathematics Education' (Ismael 2002). Marcos Cherinda's thesis (2002) deals with the mathematical-educational exploration of mat weaving patterns. Daniel Soares' thesis (2004) deals with the geometrical knowledge of house builders, in particular in the provinces of Sofala and Zambezia in the centre of Mozambique. Before they had participated in several collective studies, like the one on numeration and counting systems in Mozambique (Gerdes 1993). For instance, the following papers were published: 'The origin of the concepts of 'even' and 'odd' in Makhuwa culture (Northern Mozambique)' (Ismael), 'Popular counting practices in Mozambique' (Ismael & Soares), 'A children's 'circle of interest in ethnomathematics' (Cherinda). A third generation of Mozambicans who became interested to take part in ethnomathematical research is composed of some of our students. For instance, Salimo Saide did field work among Yao women in the north of the country, analysing the geometry of their pottery decorations (Saide 1998). Evaristo Uaile analysed some aspects of basket weaving among the Changana in the south. Gildo Bulafo did field work among Tonga women in the south-eastern province of Inhambane in order to understand better their geometrical ideas and arithmetical know-how in weaving the beautiful hand bags. Abílio Mapapá started to study the geometrical thinking of children who produce miniature wire cars. The booklet 'Explorations in ethnomathematics and ethnoscience in Mozambique' (Gerdes 1994a) presents an introduction to the work of the younger generation, including in the fields of culture and biology, physics and chemistry.

In the book 'African Pythagoras. A study in culture and mathematics education' (Gerdes 1994b) it is shown how diverse African ornaments and artefacts may be used to create a rich context for the discovery and the demonstration of the so-called Pythagorean Theorem and of related ideas and propositions. A series of earlier papers are included in the books 'Ethnomathematics: Culture, Mathematics, Education' (Gerdes 1991b) and 'Ethnomathematics and education in Africa' (Gerdes 1995a).

One of the principal lines in my own research since the end of the 1980s has been on the historical reconstruction, analysis, and educational and mathematical exploration of mathematical elements of the pictograms drawn by story tellers from the Cokwe in Eastern Angola. The book 'SONA Geometry: reflections on the tradition of sand drawings in Africa south of the Equator' (Gerdes 1993-4, 1994c, 1995c, 1997a) reconstructs mathematical components of the Cokwe drawing-illustration-tradition (Angola) and explores their educational, artistic and scientific potential. In the book 'Lusona: Geometrical recreations of Africa' (Gerdes 1991c, 1997b) mathematical amusements are presented that are inspired by the geometry of the sand drawing tradition. For children (age 10-15) the booklet 'Living mathematics: drawings of Africa' (Gerdes 1990) has been elaborated. The last part of the book 'Geometry from Africa' (Gerdes 1999) presents an introduction to Sona and Lunda geometry. The mathematical potential of Lunda designs is further explored in the book 'The beautiful Geometry and Linear Algebra of Lunda Designs' (concluded).

Another research line is the one on mathematical aspects of twill weaving in diverse cultural contexts, as attested by the comparative study 'The circle and the square: Geometric, artistic and symbolic creativity of basket weavers from Africa, the Americas, Asia and Oceania' (Gerdes 2000) and by the book 'Geometry, Symmetry and Basketry in various African and American Cultures' (Gerdes 2004).

Gerdes and Bulafo (1994) published a book on the geometrical knowledge of the mostly female weavers of the sipatsi handbags (expanded edition Gerdes 2003). This investigation of mathematical knowledge of women has been continued in the study by Gerdes (1995b, 1996a, 1998a) on women and geometry in Southern Africa, where suggestions for further research are presented.

Geometry / Mathematics in African history and cultures

The books 'Women, Art and Geometry in Southern Africa' (Gerdes 1998a), 'Geometry from Africa' (Gerdes 1999) and 'African Fractals: Modern Computing and Indigenous Design' (Eglash 1999) present overviews of geometrical ideas in African cultures. The African Mathematical Union (AMU) created in 1986 the AMU Commission on the History of Mathematics in Africa [AMUCHMA]. AMUCHMA has the following main objectives:

a. To improve communication among those interested in the history of mathematics in Africa;

b. To promote active co-operation between historians, mathematicians, archaeologists, ethnographers, sociologists, etc., doing research in, or related to, the history of mathematics in Africa;

c. To promote research in the history of mathematics in Africa, and the publication of its results, in order to contribute to the demystification of the still-dominant Eurocentric bias in the historiography of mathematics.

The AMUCHMA newsletter, published in English, French and Arabic, informs about sources on mathematical ideas in African cultures. So far, twenty-nine issues of the AMUCHMA Newsletter have been published. The English language edition of the AMUCHMA-Newsletter is available on-line (www.math.buffalo.edu/mad/AMU /amuchma_online.html). At the 6th Pan-African Congress of Mathematicians held in Tunis (September 2004) the AMU launched the annotated bibliography '*Mathematics in African History and Cultures*' (Gerdes & Djebbar 2004), with over a thousand references to studies on mathematical ideas in Africa's history from immortal times to the present, including references on the integration of indigenous/endogenous mathematical knowledge into mathematics education.

Integration of ethnomathematics into teacher education: Social- and cultural-mathematical awareness

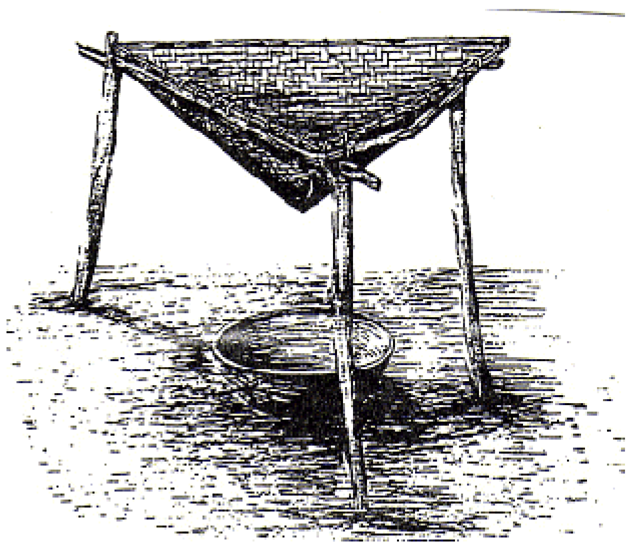
In this contribution to 'All knowledge is first of all local knowledge', I would like to include some of my personal experiences with bringing African knowledge and wisdom into the university mathematics classroom. I will summarise some examples presented earlier in the paper 'Developing social- and cultural-mathematical awareness in mathematics teacher education in a multicultural African context (Mozambique)' (Gerdes 1998b). The first example of a dialogue between a teacher educator and his students dates back from one of the first quick one-year mathematics teacher education programmes, early after Mozambique's independence in 1975, at the Eduardo Mondlane University at the end of the 1970s.

First example

Basket weavers from the North of Mozambique produce a pyramidal funnel (see Figure 1), called 'eheleo' in the Makhuwa language. To do so, they start with weaving a square mat, completing it unto the middle and then inter-weaving the two halves of outstanding strand parts. As a result, the funnel's mouth has the form of an equilateral triangle. In the 'Geometry' course I taught at the time I displayed an 'eheleo' to my students and asked: 'What can we learn from the artisans, from their production technique?' 'May it suggest us a method to construct equilateral triangles?'

Students reacted rather sceptically: 'It seems a very clumsy method to do so ...'. But the objective of the artisan was to produce a funnel, and not to construct an equilateral triangle. Can we adapt the artisans' method to serve our purpose? For instance, how can we transform a square of cardboard paper into a pyramidal funnel?

Figure 1: Makhuwa woven funnel ('eheleo')



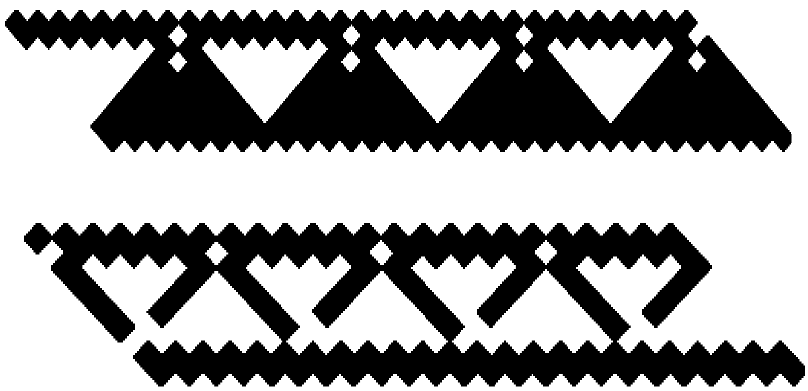
The students found a way to fold the cardboard square in such a way that it transforms into a pyramid. Then we organised a little competition. One group of students had to construct equilateral triangles with ruler and compass, while another group by folding a square. Some students were quicker using the first method, others using the second. The point was made. We all could learn from those Makhuwa basket weavers from the North. Then we advanced, posing another question: Is it possible to generalise the 'eheleo' method. Indeed this is possible, and the students found out that by folding a regular octagon in the same way as the square, that then a regular heptagon could be constructed. By folding once more, a regular hexagon appears. More, in principle all regular polygons may be constructed in a similar way by starting with (easy to fold) regular 8-, 16-, 32-, 64-, 128-gons (etc.). General surprise emerged among the students. Had not the German Gauss proven that many regular polygons, including regular heptagons, are impossible to construct with ruler and compass? The students arrived at the conclusion that what is possible to construct depends on which are the means that are available and that are admitted. They understood that admission implies a choice that is culture-dependent. They understood that it is possible to discover new (strong) construction methods by reflecting on cultural elements of the country. All students felt proud of the 'eheleo' method for constructing regular polygons. And as one of them observed 'Not all mathematical ideas come from the 'West!' In other words, their cultural-mathematical self-confidence had risen.

The following examples are from experiences at the Universidade Pedagógica (UP), founded in 1986. This university prepares teachers and other educational specialists, like educational planners and educational psychologists, in 4 to 5 year 'licenciatura' programmes. It started in Maputo and established branches in Beira (centre of the country, 1989), Nampula (north of the country; 1995), and Quelimane (central north-east, 2003). One of the objectives of the obligatory course '*Mathematics in History*' for the students in Mathematics Education programme is to contribute to a broader historical, social and cultural perspective on and understanding of mathematics. The first theme '*Counting and Numeration Systems*' gives a good start, as the students can begin with analysing and comparing together the various ways of counting and numeration they learned in their life, discovering the rich variety at the national level. Thereafter, they are brought into contact with systems both from other parts of Africa and the world, and from other historical periods. The introduction of optional courses in 1993 gave my colleagues and me, the chance to introduce courses like '*Culture, Symmetry and Geometry' and* '*Ethnomathematics and the Teaching of Mathematics in Secondary and High Schools*' with a strong cultural component. Before the introduction of optional courses (for which the students receive credits), we experienced with 'circles of interest' or clubs, in which students (and interested lecturers) take voluntary part, focussed on a general theme of 'Ethnomathematics' or on more specific themes like 'Mathematical and educational exploration of basket weaving techniques', 'Geometry of African sand drawings', and 'Lusona - African Geometrical Recreations'. For taking part in these 'circles of interest' the students receive 'diplomas of participation'. Those students who showed particular interest in the themes analysed in the optional courses or 'circles of interest' have been invited to accompany 'children's clubs' (e.g. on the theme 'Thousand and one beautiful weaving designs', directed by Marcos Cherinda) and/or to do fieldwork in their home regions. All participants have been stimulated to develop specific (sub) themes for experimentation in secondary or high schools. The realisation of the optional courses, 'circles of interest' and fieldwork constitutes both part of the research integrated in our Ethnomathematics Research Project and a possibility to develop (and reflect on this development) social- and cultural-mathematical awareness, as the following examples may illustrate.

Second example

One theme in an optional course I gave on '*Culture, Symmetry and Geometry*' (1994) for fourth year students was the geometry of the weaving of the 'sipatsi' bags among the Gitonga speaking population in Inhambane province. The making of 'sipatsi' with their band decorations (see the examples in Figure 2) is traditionally a female domain, although more recently also some men learned to weave them.

Figure 2: Examples of decorative bands on 'sipatsi' bags



In the course, there were only two female students. When the theme of the 'sipatsi' came up, they appeared more sceptical than their male colleagues: 'Those basket weavers do not apply mathematics', suggesting to advance with some more 'modern' topic. After analysing together how important it is, before starting to weave, to take into account the periods[1] in order to get good quality 'sipatsi', where on the cylindrical wall each decorative motif appears exactly a whole number of times, the female student who had been more reluctant to accept the 'sipatsi' theme, remarked that she did not believe that the basket weavers were capable of doing the necessary mental calculations; it was only 'good luck' or 'intuition'. Her ideas began to change, when she started to analyse herself some beautifully decorated 'sipatsi': in the case of the combination of decorative bands in Figure 3, the total number of plant strands in each of the two weaving directions has to be a common multiple of the two periods (3 and 10), that is of 30, and this number has to be known before starting the weaving, as it is impossible to increase or decrease later on the number of plant strands; now she realised that starting with 'good luck' or 'intuition' really did not 'do the job'.

Figure 3: Combination of two different bands on the same 'sipatsi' bag

Once increased her interest in the geometry of the 'sipatsi', she started the work with enthusiasm and fervour on enumeration and generation problems I proposed to the students: How many possible band patterns (of the 'sipatsi' type) of given dimensions *p* and *d* do exist, whereby *p* denotes the period of the respective decorative motif and *d* its 'diagonal height?' She was the first to find several solutions and she explained proudly to her colleagues her results and the reasoning that led to them. Coming from another region of the country, grown up in the capital, her appreciation of the craft and knowledge of the female basket weavers had changed radically, and she showed more confidence in her own capacities to obtain new results.

The next two examples consist of the testimonies of two graduates of the 'Universidade Pedagógica', both from the north of the country.

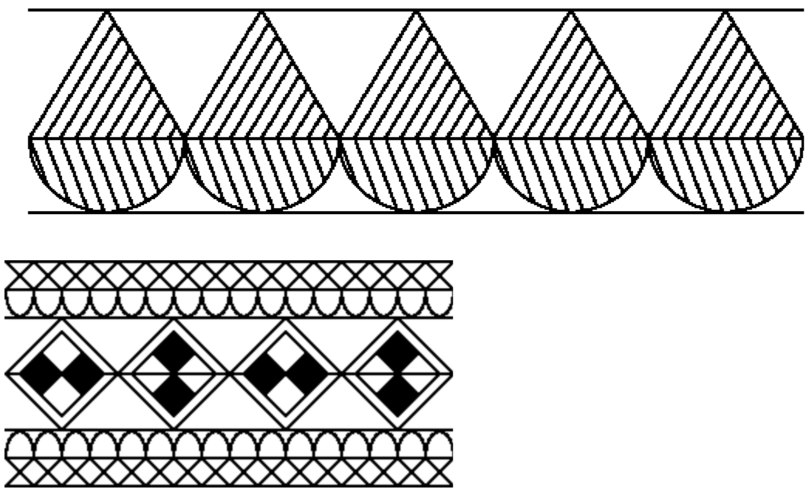
Third example: Testimony of Salimo Saide

I was born June 20, 1965 in Lichinga, capital of the northern Niassa Province. There I went to primary and secondary school. From 1985 to 1987 I took part in a teacher education program. From 1987 to 1991 I taught Mathematics and Physics at the secondary school of Pemba, capital of the Cabo Delgado Province where I co-ordinated the local Mathematics Olympiads. In 1991 I came to Maputo to continue my studies and in 1996 I concluded my 'Licenciatura' in Mathematics and Physics at the 'Universidade Pedagógica'.

In 1977 I had the opportunity to read a book written by the priest Yohana, entitled 'Wa'yaowe', that means 'We the Yao people'. It opened a whole new horizon for me. I was lucky to be able to read and write Yao - in school only Portuguese is taught. When I came to Maputo I thought my dream had died. However, when I took part first in a voluntary 'circle of interest' on mathematical elements in African cultures and then in the optional course

'Ethnomathematics and Education' my dream started to live again. I found a strong link between mathematics and the art of my grandparents. My participation let me return to my land, let me remember my grandmother, her decorated mats and baskets, and her beautiful 'nembo' – tattoos and pot decorations (see Figure 4). The idea 'caught' me and during my holidays I made three field trips to Niassa to study the geometry of ceramic pot ornamentation. Now after finishing my studies, I hope to return to my land, to continue my research and to teach mathematics integrating the 'nembo' of the Yao people into it' (cf. Saide 1998).

Figure 4: Examples of 'nembo' strip decorations on pots



It was not easy for Salimo to realise his fieldwork. Sometimes it took him various encounters on several successive days to win the confidence of the old female pot makers, as they did not understood easily why a young man, speaking with the accent of someone educated in the cities, could be interested in their nowadays downgraded and disappearing female art and craft of pot decoration; why would he be interested to see their tattoos when the churches, both Christian and Islamic, have been combating tattooing so strongly? However, once he won their confidence, they were happy to speak about their craft and art, and about how they learned it, to discuss with the student alternative ways of reviving, of valuing their symbolic language, their knowledge, wisdom, and creativity. For instance, it was suggested to decorate 'capulanas' - square woven cloths worn by the women around their middle - with ceramic 'nembo' and T-shirts with tattoo 'nembo'.

Fourth example: testimony by Abel Tomo

I was born on June 26, 1970 in Cuamba in the Makhuwa speaking part of the Niassa Province. My father and mother are peasants. In their leisure time, my father weaves colourful baskets, and my mother makes decorated pots. While in primary school, I began to ask myself how could my parents be able to make such beautiful objects without having been to school; they even did not know mathematics. I felt a strong contradiction between school and home, particularly in the mathematics lesson. For secondary school I went to the Nampula province. There I took part in my leisure time in a 'circle of interest' organised by an archaeologist. In the course of helping him I started to understand the history and culture of the Makhuwa, but still did not understand the relationship with mathematics. I could not solve the contradiction. Five years ago I came to Maputo to study Mathematics and Physics at the 'Universidade Pedagógica'. Through the course 'Mathematics in History' I got some ideas to reflect about, but the optional course 'Ethnomathematics and the teaching of mathematics' really opened my eyes. I wanted to do field work among my people, and in December 1995 – January 1996 I went to Niassa and Nampula, and learned a lot from older peasants about how they fabricate beautiful objects. I learned from boys and girls in the villages how to make several toys. And so I began little by little to understand the geometry of my parents. Having now finished my university program I will return to my people, trying to value its knowledge in my teaching.

Teachers like Salimo and Abel - who as students voluntarily took part in 'clubs' and optional courses related to culture and mathematics education - return, well motivated, to their home provinces, determined to work as mathematics teachers in such a way that it is both useful for their people and dignifying to its cultural heritage.

Fifth example

Marcos Cherinda is a native Ronga speaker from the south of Mozambique. During several years he lived with his sister, a nurse, in Nampula in the north, becoming fluent in Makhuwa. At secondary school he took part in various clubs related to the culture of Nampula, and exploring his drawing talents he elaborated a band strip on Nampula's history. In 1980 he came to Maputo to take part in an accelerated two-year teacher education program at the Eduardo Mondlane University. As one of my students he showed much interest in cultural aspects of mathematics (education), and I invited him to write a paper on circles in Makhuwa culture for '*Tlanu*', the Mozambican Journal on Mathematics Education. He contributed a paper on the use of the circle concept among fishermen from Nampula province (see one of his illustrations, reproduced in Figure 5). After two years of teaching in a secondary school, he went to Europe to do M.Ed. in mathematics education. Upon returning in 1989, he was recruited as an assistant lecturer at the 'Universidade Pedagógica', teaching geometry and integrating himself into the Ethnomathematics Research Project, and he did fieldwork in the Maputo and Inhambane provinces. He took part in various 'circles of interest' and co-operated in the optional courses given. Since a few years he organises 'children's clubs', integrating students, exploring, in particular, possibilities to use a weaving board to develop geometrical knowledge - the theme of his Ph.D. thesis 'The use of a cultural activity in the teaching and learning of mathematics: The exploration of twill weaving in Mozambican classrooms' (Cherinda 2002).

Figure 5: Drying fish in a circle on Mozambique's island (Nampula Province)

Experimentation with ideas from ethnomathematics in education

Ethnomathematical and historical research clearly shows that mathematical education did not start in Africa with the arrival of the 'white man' to the shores of Africa. Ethnomathematical research findings urge to reflect about fundamental mathematical-educational questions: Why teach mathematics? What and whose mathematics should be taught, by whom and for whom? Who participates in curriculum development? etc.

In the final section of this paper three examples of (complementary and partially overlapping) trends in using ideas from ethnomathematics in education in Africa, will be briefly presented.

The first example illustrates an experience with the incorporation into the curriculum of elements belonging to the socio-cultural environment of the pupils and teachers, as a starting point for mathematical activities in the classroom, increasing the motivation of both pupils and teachers.

Example: Cowry games in Côte d'Ivoire

In 1980 a research-seminar on 'Mathematics in the African socio-cultural environment' was introduced at the Mathematical Research Institute of Abidjan (IRMA, Côte d'Ivoire). Salimata Doumbia directs the seminar. One of the interesting themes analysed by her and her colleagues is the mathematics of traditional West-African games. Their work deals with classification of the games, solution of mathematical problems of the games and exploring the possibilities of using these games (e.g. Nigbé Alladian) in the mathematics classroom.

One plays Nigbé Alladian with four cowry shells. On their turn, each of the two players casts the cowry shells. When all four land in the same position, i.e. all 'up' or all 'down', or when two land in the 'up' position and the other two in the 'down' position, the player gets points. In the other cases, one 'up' and three 'down', or, three 'up' and one 'down', a participant does not get points. As the researchers of IRMA found experimentally that the chance of a cowry shell to fall in the 'up' position is 2/5, it came out that the rules of the game had been chosen in such a way that the chance to win points is (almost) the same as to get no points. Doumbia concluded 'without any knowledge of calculation of probability, the players have managed ...to adopt a clever counting system, in order to balance their chances' (Doumbia 1989). This and other games are embedded into the secondary school curriculum as an introduction to probability theory and computer simulation. Interesting examples are given in the book by Doumbia & Pil (1992).

The next example presents an experience with the conscientization of future mathematics teachers and teacher educators of the existence of mathematical ideas similar to or different from those in the textbooks among people with little or no formal education; learning to respect and to learn from other human beings, possibly belonging to other social/cultural (sub)groups.

Example: Market women in Mozambique

Lecturers and students of the 'licenciatura' Programme in Mathematics Education for Primary Schools at the Beira Branch of Mozambique's Universidade

Pedagógica have been analysing arithmetic in and outside school. On interviewing illiterate women to know how they determine sums and differences, it was found that the women 'solved easily nearly all the problems, using essentially methods of oral/mental computation, i.e., computation based on the spoken numerals. The methods used were very similar to those suggested by the present day mathematics syllabus for primary education, but including some interesting alternatives' (Draisma 1992). For instance, 59% of the interviewed women calculated mentally 62-5 = ..? by first subtracting 2 and then 3, i.e. they used the same method as is emphasised in the schoolbook. Another 29% of the women subtracted first 5 from 60 and then added 2, and 12% subtracted first 10 from 62, and added the difference between 10 and 5, i.e. 5.

Did these women *(re)invent* their method? Did they learn them? From whom and how?

When multiplying, most of the interviewed women solve the problems by doubling. An example illustrates the process 6 x 13 = ..? Schematically the solution is the following: 2 x 13 = 26; 4 x 13 = 2 x 26; 2 x 26 = 52; 6 x 13 = 26 + 52; 26 + 52 = 78. Does each of these women (re)invent the doubling method spontaneously? Or does there exist a tradition? If so, how is the method taught and learnt?

The last example illustrates the preparation of future mathematics teachers to investigate mathematical ideas and practices of their own cultural, ethnic, linguistic communities and to look for ways how to build upon them in their teaching.

Example: Peasants in Nigeria

Shirley (1988) and his students at the Ahmadu Bello University in Nigeria conducted oral interviews with unschooled, illiterate members of the students' home communities. They found that 'although some of the (arithmetical) algorithms used by the informants are similar to those taught in schools, some interesting non-standard techniques were also found'. Shirley advises that one should assign teacher-student to find (ethno) algorithms in their communities - literate or illiterate, rural or urban, as 'Too often, school lessons leave the impression that there is only one way to do a given task'.

Note

1. Period = the number of plant strands in one direction to generate a copy of the decorative motif. In the examples in Figure 7, they are 6, 8 and 8 respectively.

References

D'Ambrosio, Ubiratan, 1990, *Etnomatemática: arte ou técnica de explicar e conhecer*, Editora Ática, São Paulo.

Cherinda, Marcos, 2002, 'The Use of a Cultural Activity in the Teaching and Learning of Mathematics: The Exploration of Twill Weaving in Mozambican Classrooms', Unpublished Ph.D. thesis, Witwatersrand University, Johannesburg.

Doumbia, Salimata,1984, *Mathématiques dans l'environnement socio-culturel Africain*, Abidjan : Institut de Recherches Mathématiques d'Abidjan.

Doumbia, Salimata, 1989, 'Mathematics in traditional African games', in: C. Keitel, P. Damerow, A. Bishop, P. Gerdes (eds.), *Mathematics, Education, and Society*, Paris: UNESCO, 174-175.

Doumbia, Salimata & Pil, J.,1992, *Les jeux de cauris*, Abidjan: IRMA.

Draisma, Jan, 1992, 'Arithmetic and its didactics', in *Report of the first year of the Master's Degree Program in Mathematics Education for Primary Schools* (August 1991–July 1992), Instituto Superior Pedagógica, Beira, 1992, 89-129 (mimeo).

Eglash, Ron, 1999, *African Fractals: Modern Computing and Indigenous Design*, Piscataway: Rutgers University Press.

Gay, John & Cole, Michel, 1967, *The new mathematics and an old culture, a study of learning among the Kpelle of Liberia*, New York: Holt, Rinehart & Winston.

Gerdes, Paulus, 1982, *Mathematics for the benefit of the people*, CARIMATH, Paramaribo (mimeo).

Gerdes, Paulus, 1985a, *Conditions and strategies for emancipatory mathematics education in underdeveloped countries*, For the Learning of Mathematics, Montreal, 5(3), 15-20.

Gerdes, Paulus, 1985b, *Zum erwachenden geometrischen Denken*, Maputo: Eduardo Mondlane University (mimeo).

Gerdes, Paulus,1990, *Vivendo a matemática: desenhos da África*, São Paulo: Editora Scipione.

Gerdes, Paulus, 1991a, *Ethnogeometrie. Kulturanthropologische Beiträge zur Genese und Didaktik der Geometrie*, Verlag Franzbecker, Bad Salzdetfurth, 360 p. (new edition 2003).

Gerdes, Paulus, 1991b, *Etnomatemática: Cultura, Matemática, Educação*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus, 1991c, *Lusona: geometrical recreations of Africa*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus, ed.,1993, *A numeração em Moçambique - Contribuição para uma reflexão sobre cultura, língua e educação matemática*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus,1993-1994, *Geometria Sona: Reflexões sobre uma tradição de desemho em povos da África ao Sul do Equador*, Maputo: Instituto Superior Pedagógico, (3 vol.).

Gerdes, Paulus, ed.,1994a, *Explorations in Ethnomathematics and Ethnoscience in Mozambique*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus, 1994b, *African Pythagoras: A study in culture and mathematics education*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus, 1994c, *SONA Geometry: Reflections on the tradition of sand drawings in Africa south of the Equator*, Maputo: Instituto Superior Pedagógico, Vol. 1.

Gerdes, Paulus, 1995a, *Ethnomathematics and Education in Africa*, Stockholm: University of Stockholm Institute of International Education.

Gerdes, Paulus, 1995b, *Women and Geometry in Southern Africa, Suggestions for further research*, Maputo: Universidade Pedagógica.

Gerdes, Paulus, 1995c, *Une tradition géométrique en Afrique: Les dessins sur le sable*, Paris: L'Harmattan, (3 vol.).

Gerdes, Paulus, 1996a, 'Ethnomathematics and mathematics education: an overview', in: A. Bishop (ed.), *International Handbook of Mathematics Education*, Kluwer, Dordrecht, 909-944.

Gerdes, Paulus, 1996b, *Femmes et Géométrie en Afrique Australe*, Paris: L'Harmattan.

Gerdes, Paulus, 1997a, *Ethnomathematik dargestellt am Beispiel der Sona Geometrie*, Heidelberg: Spektrum Verlag.

Gerdes, Paulus, 1997b, *Récréations géométriques d'Afrique - Lusona - Geometrical recreations of Africa*, Paris: L'Harmattan, 127 p. (Bilingual edition in French and English).

Gerdes, Paulus, 1998a, *Women, Art and Geometry in Southern Africa*, Lawrenceville NJ / Asmara: Africa World Press.

Gerdes, Paulus, 1998b, 'On culture and mathematics teacher education', *Journal of Mathematics Teacher Education,* Dordrecht (Netherlands), Vol. 1, No. 1, 33-53.

Gerdes, Paulus, 1999, *Geometry from Africa: Mathematical and Educational Explorations*, Washington DC: The Mathematical Association of America.

Gerdes, Paulus, 2000, *Le cercle et le carré: Créativité géométrique, artistique, et symbolique de vannières et vanniers d'Afrique, d'Amérique, d'Asie et d'Océanie*, Paris: L'Harmattan.

Gerdes, Paulus, 2003a, *Awakening of Geometrical Thought in Early Culture*, Minneapolis MN: MEP Press.

Gerdes, Paulus, 2003b, *Sipatsi: Cestaria e Geometria na Cultura Tonga de Inhambane*, Maputo: Moçambique Editora.

Gerdes, Paulus, 2004, *Geometry, Symmetry and Basketry in various African and American Cultures*, Special book edition of Visual Mathematics, Belgrad (forthcoming).

Gerdes, Paulus & Bulafo, Gildo,1994, *Sipatsi: Technology, Art and Geometry in Inhambane*, Maputo: Instituto Superior Pedagógico.

Gerdes, Paulus & Djebbar, Ahmed, 2004, *Mathematics in African History and Cultures. An Annotated Bibliography*, Cape Town: African Mathematical Union.

Ismael, Abdulcarimo, 2002, 'An ethnomathematical study of Tchadji: About a Mancala type board game played in Mozambique and possibilities for its use' in *Mathematics Education*, Unpublished Ph.D. thesis, University of Witwatersrand, Johannesburg.

Julie, Cyril, ed., 1989, *Proceedings of a Conference on the Politics of Mathematics Education*, NECC Mathematics Commission, Cape Town.

Kane, Elimane, 1987, *Les systèmes de numération parlée des groupes ouest-atlantiques et Mande. Contribution à la recherche sur les fondaments et l'histoire de la pensée logique et mathématique en Afrique de l'Ouest*, Unpublished PhD thesis, University of Lille, Lille.

Posner, Jill, 1982, 'The development of mathematical knowledge in two west African societies', in *Child Development,* Chicago, Vol.53, 200-208 [based on her Ph.D. thesis, 1978].

Raum, Otto, 1938, *Arithmetic in Africa*, London: Evans Brothers.

Saide, Salimo, 1998, *On the Geometry of Pottery Decoration by Yao Women*, in: (Gerdes, 1998a), 203-230.

Shirley, Lawrence, 1988, *Historical and ethnomathematical algorithms for classroom use*, Zaria: Ahmadu Bello University, (mimeo).

Soares, Daniel, 2004, *The incorporation of the geometry involved in traditional house building in Mathematics Education in Mozambique. The cases of the Zambezia and Sofala Provinces*, Unpublished Ph.D. thesis, University of the Western Cape, Rondebosch.

Zaslavsky, Claudia, 1973, *Africa Counts: number and pattern in African culture,* Prindle, Weber and Schmidt, Boston (new edition: Lawrence Hill Books, Brooklyn, NY, 1979, 1999).

Zaslavsky, Claudia, 1994, 'Mathematics in Africa: Explicit and implicit', in Grattan-Guinness, Ivor (ed.): *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, London: Routledge, Vol. 1, 85-92.